

SLICING AND DICING THE MILKY WAY DISK IN THE SLOAN DIGITAL SKY SURVEY

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ABSTRACT

We use the Stripe 82 proper motion catalogue of Bramich et al. (2008) to study the kinematics of Galactic disk stars in the solar neighborhood. We select samples of dwarf stars with reliable spectra and proper motions. They have cylindrical polar radius between $7 \leq R \leq 9$ kpc, heights from the Galactic plane satisfying $|z| \leq 2$ kpc and span a range of metallicities $-1.5 \leq [\text{Fe}/\text{H}] \leq 0$. We develop a method for calculating and correcting for the halo contamination in our sample using the distribution of rotational velocities. Two Gaussians representing disk and halo populations are used to fit the radial (v_R) and vertical (v_z) velocity distributions via maximum likelihood methods. For the azimuthal velocities (v_ϕ) the same technique is used, except that a skewed non-Gaussian functional form now represents the disk velocity distribution. This enables us to compute the dispersions $\sigma_R, \sigma_z, \sigma_\phi$ and cross-terms (the tilt σ_{Rz} and the vertex deviation $\sigma_{R\phi}$) of the velocity ellipsoid as a function of height and metallicity. We also investigate the rotation lag of the disk, finding that the more metal-poor stars rotate significantly slower than the metal-rich stars. These samples provide important constraints on heating mechanisms in the Galactic disk and can be used for a variety of applications. We present one such application, employing the Jeans equations to provide a simple model of the potential close to the disk. Our model is in excellent agreement with others in the literature and provides an indication the disk, rather than the halo, dominates the circular speed at the solar neighborhood. We obtain a surface mass density within 1.1 kpc of around $66 \text{ M}_\odot \text{pc}^{-2}$ and estimate a local halo density of $0.015 \text{ M}_\odot \text{pc}^{-3} = 0.57 \text{ GeV cm}^{-3}$.

Subject headings: Galaxy: disk — Galaxy: evolution — Galaxy: kinematics and dynamics — solar neighborhood

1. INTRODUCTION

In the simplest model of a spiral galaxy, stars in the disk librate about circular orbits in the equatorial plane (“the epicyclic approximation”, Binney & Tremaine 2008). This picture is complicated by astrophysical processes: such as bar instabilities, satellite accretion, scattering by spiral arms or collisions with molecular clouds. Surveys which gather kinematic data allow astronomers to analyze the properties of large samples of stars and hence probe the signatures left by heating processes in their net motions and dispersions. For example, the *Hipparcos* satellite provided precise parallaxes and proper motions for nearby stars, which were used to provide the velocity dispersion as a function of stellar type in the immediate solar neighborhood (Dehnen & Binney 1998a).

In their 1983 study of the density distribution of stars up to 4 kpc below the plane, Gilmore & Reid argue that the disk is in fact composed of two distinct components: a thin disk with approximate scale height ~ 300 pc, and an older, more metal-poor thick disk. Further studies (see Reid & Majewski 1993 for a review) have supported this claim, although the thick disk scale height remains poorly constrained (eg. Jurić et al 2008; De Jong et al. 2010) and the question of how the two-component structure arose is still open. Suggested scenarios include reso-

nant trapping, which converts planar orbits into inclined ones (Sridhar & Touma 1996), or the dissipational collapse of gas (Brook et al. 2004), or the accretion of a satellite galaxy, in which case the thick disk may form from satellite debris, or as a result of heating of the thin disk by the collision (eg. Villalobos & Helmi 2008). In recent years a lot of attention has focussed on the formation of thick disks through radial migration, where stars trapped in co-rotation resonances of the spiral arms are transported through the disk (Sellwood & Binney 2002; Roškar et al. 2008; Schönrich & Binney 2009) or through similar mechanisms due to the Galactic bar (Minchev & Famaey 2010). Detailed tests of the chemical properties of the Milky Way disk are also becoming possible, for example using large samples of stars with alpha-element abundances to characterize the origins of the different components (e.g. Bovy et al. 2011; Lee et al. 2011; Navarro et al. 2011; Ruchti et al. 2010).

Much can also be learnt about heating processes through studying the size and shape of the stellar velocity ellipsoid, or equivalently the stellar velocity dispersion tensor. The ratio of radial to vertical velocity dispersion σ_R/σ_z , as well as the dependence of the dispersion on the age of a stellar population, can be compared with analytical and numerical predictions for various models of disk heating, to see which mechanisms dominate in our galaxy (Fuchs & Wielen 1987). The ratio of tangential to radial velocity dispersion σ_ϕ/σ_R is predicted to be $1/\sqrt{2}$ from epicyclic theory for a galaxy with a flat rotation curve, though it has long been known to be less than this for the Milky Way (Kerr & Lynden-Bell 1986; Evans & Collett 1993). The covariances σ_{Rz} and $\sigma_{R\phi}$

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are also of interest; non-zero $\sigma_{R\phi}$ implies that the Galactic potential is non-axisymmetric (Binney & Tremaine 2008). The quantity σ_{Rz} , or more specifically, its vertical gradient, is important for the calculation of Galactic parameters, including the local surface mass density of the disk (Kuijken & Gilmore 1989) and the asymmetric drift relation (Dehnen & Binney 1998a).

Large-scale photometric and kinematic surveys are making measurements of the components of the velocity dispersion tensor possible for populations in the Milky Way Galaxy. For example, using data from the Radial Velocity Experiment (RAVE), Siebert et al. (2008) measured the magnitude and orientation of the velocity dispersion tensor for red clump stars between 500 pc and 1500 pc below the Galactic Plane. They reckoned the tilt to be $7.3 \pm 1.8^\circ$, which is consistent with alignment in spherical polar coordinates. Siebert et al. compare this value to computed inclinations for two mass models of the Milky Way. The measurement is reproducible with a short scalelength of the stellar disk (≈ 2 kpc) if the dark halo is oblate or with a long scalelength (≈ 3 kpc) if the dark halo is spherical or prolate. A similar study has been published subsequently by Casetti-Dinescu et al. (2011), also using radial velocities from the RAVE survey, finding a tilt of $8.6 \pm 1.8^\circ$ for red clump stars at a distance of 700 to 2000 pc from the plane.

The kinematics properties of the Galactic disk can also be used to probe directly its gravitational potential, as was carried out in the classical work of Kuijken & Gilmore (1991). Using a sample of around 500 K dwarfs, they were able to estimate that the surface mass density within 1.1 kpc was $71 \pm 6 \text{ M}_\odot \text{pc}^{-2}$. Subsequent work has supported this measurement, for example, Holmberg & Flynn (2004). An inventory of the surface density of baryonic material in the solar neighborhood indicates that around a third of this mass is likely non-baryonic (e.g. Binney & Evans 2001). Such techniques can also be used to determine the local mass density and hence make predictions for the local dark matter density (e.g. Holmberg & Flynn 2000; Garbari et al. 2011), which is a vital ingredient in predictions for the direct detection of dark matter (Bertone, Hooper & Silk 2005, and references therein).

Clearly obtaining unbiased determinations of the kinematic properties of the disk, including their gradients, is of great importance for understanding its structure and evolution. Here, we study the kinematics of disk stars using data from the recently constructed Sloan Digital Sky Survey (SDSS) Stripe 82 catalogue (Bramich et al. 2008). This covers an equatorial stripe of area 250 deg^2 , which has been repeatedly imaged by the SDSS since 1998 primarily with the aim of supernova discovery. The catalogue contains almost 4 million ‘‘light-motion curves’’ and is complete down to a magnitude of $r \approx 21.5$. It reaches almost 2 mag fainter than the SDSS/USNO-B catalogue (Munn et al. 2004), making it the deepest large-area photometric and astrometric catalogue available. We have already exploited the Stripe 82 catalogue to measure the components of the velocity dispersion tensor for a sample of halo stars (e.g. Smith et al. 2009a,b). Here, we provide a complementary study for disk populations.

In Section 2, we describe the selection cuts that we apply to the Stripe 82 catalogue to generate our sample

of dwarf stars. The procedure for splitting the sample into disk and halo components is described in Section 3, as well as a discussion of the effects of measurement errors. Our results are presented and discussed in Section 4.

2. SAMPLE CONSTRUCTION

We use data from the 7th public data release of the Sloan Digital Sky Survey (SDSS; Abazajian et al. 2009; Yanny et al. 2009). We restrict attention to stars in the spectroscopic part of the survey, so as to have estimates of velocities, metallicities and surface gravities. The full details describing the analysis of these data, in particular the reliability of the derived velocities and stellar parameters, can be found in Lee et al. (2008) and references therein.

Although SDSS data cover a huge part of the sky, mainly in the Northern Galactic cap, we also restrict ourselves to data in Stripe 82, so as to exploit the catalogue of high-precision photometry and proper-motions (Bramich et al. 2008). This stripe covers around 250 square degrees in region $\alpha = 20.7 \text{ hr}$ to 3.3 hr and $|\delta| < 1.26^\circ$. This corresponds to the South Galactic cap, with $l \sim 50$ to 190 degrees and $b \sim -25$ to -60 degrees.

From this data, we select stars with reliable spectra (i.e. labelled in the catalogue by the flag ‘nnnn’) and errors in $\log g$ and $[\text{Fe}/\text{H}]$ of less than 0.5 dex and in radial velocity of less than 20 km s^{-1} . We require proper motion errors of less than 4 mas yr^{-1} . We also place a cut in $\log g$ in order to remove giant stars from the sample, which we place at $\log g = 3.5$. To investigate the effects of giant contamination, we repeat our analysis of the sample properties using the stricter selection criterion $\log g \geq 4$. This stricter cut excludes only ~ 1 per cent of our sample. We checked that this made no qualitative differences to the trends observed.

The errors quoted in the SDSS stellar parameter pipeline are internal errors. Comparison with observations made using high-resolution spectroscopy reveals that there are also external errors of 2.4 km s^{-1} , 0.11 dex and 0.21 dex present in the radial velocities, metallicities and surface gravities, respectively (Allende Prieto et al. 2008). We add the internal and external errors in quadrature to obtain the overall error in each measurement.

We correct all magnitudes for the effects of extinction by interstellar dust, using the maps of Schlegel et al. (1998). We also place a cut in color of $0.3 \leq (g-i) \leq 4.0$. This is required so that we can obtain reliable photometric distances, which we estimate using the relation of Ivezić et al. (2008, equations A1-A5). We make one minor modification to the relation of Ivezić et al. (2008), namely we do not apply the turn-off correction given by equation (A6) of their paper. Their correction is based on the metal-poor globular cluster M13, which may not be suitable for disk stars. In Appendix A we use stellar models to show that a better choice may be to simply neglect this correction term. Note that although Ivezić et al. (2008) use photometric estimates for the metallicities in their paper, we are able to use the more reliable spectroscopic metallicities for our sample. It is worth pointing out that although the Ivezić et al. (2008) paper deals with the analysis of photometric metallicities, their parallax relation is derived from star clusters; as a con-

sequence our work will not be affected by any systematic offsets between spectroscopic and photometric metallicities (should any be present). The median error on our distances is 10.7 per cent.

Once we have an estimate for the distance, we can determine the full three-dimensional positions and velocities. We express these in terms of Galactocentric cylindrical coordinates, (R, ϕ, z) , where ϕ is defined as increasing in the direction opposite to the solar rotation, and z is positive towards the North Galactic Pole (NGP). We calculate corresponding velocities (v_R, v_ϕ, v_z) for each star, and correct for the solar motion (Dehnen & Binney 1998a) and the motion of the local standard of rest (LSR) so that our stellar velocities are relative to the Galactic frame, ie. $v_{\text{LSR}} = -220 \text{ km s}^{-1}$.

The errors on the positions and velocities are calculated using a Monte Carlo method. This is done by taking the uncertainties on all of the observed quantities for a given star (proper motion, radial velocity, magnitudes and metallicity) and randomly sampling from these assuming the errors are Gaussian; for each realisation we then calculate a distance (including uncertainties in the photometric parallax relation) and hence a velocity. We repeat this sampling 1000 times and from the resulting distributions of positions and velocities calculate our error matrix for a given star. The median values of the errors δv_R , δv_ϕ and δv_z are 27.2, 27.3 and 16.9 km s^{-1} , respectively. As our line of sight is approximately aligned with the South Galactic Pole, v_z is mostly determined by radial velocity measurements, whereas v_R and v_ϕ are mostly dependent on proper motions. The measurement errors in the proper motions are much larger than in the radial velocities, hence δv_R and δv_ϕ are larger than δv_z . We also calculate the correlations in the errors (e.g. $\delta v_R v_z$), which are important for the covariances.

3. ALGORITHMS FOR DISK KINEMATICS

Here, we describe the techniques used to obtain the kinematic properties of the disk. We aim to study the properties of our sample as a function of height above or below the Galactic plane z . However, many of these properties depend strongly on metallicity, so to allow for the influence of vertical metallicity gradients in the disk, we split the sample into ranges in metallicity and study trends in each range separately. This is also crucial because the SDSS spectroscopic selection function can introduce biases which are difficult to model. Therefore, by binning our data in metallicity ranges, we are only making the assumption that the metallicity distribution *within a bin* is representative of the true metallicity distribution. Although this will not be exactly the case due to variations within a bin, both in terms of metallicity and age, it should remove the main source of systematic bias in the analysis.

The range of our data in cylindrical polar radius R is too narrow for us to determine trends in the radial direction with any confidence. Clearly, the further we go from the plane, the larger the range in R which is covered, and so in order to keep our data to a specific range in R we restrict ourselves to $7 \leq R \leq 9$. Therefore, we split the data into three ranges in metallicity ($-1.5 \leq [\text{Fe}/\text{H}] \leq -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $-0.5 \leq [\text{Fe}/\text{H}]$), and then for each metallicity bin we further divide the data into four ranges in z out to a

maximum distance of 2 kpc. A total of 7280 stars match these criteria. The stars are equally divided between the four distance bins, resulting in around 500 to 800 stars per bin (see Table 1).

3.1. The Stellar Halo Contribution

In order to carry out an unbiased study of the kinematics of the disk, we need a method to model the contamination from the halo stars in our sample. To do this, we make the assumption that all counter-rotating ($v_\phi > 0$) stars belong to the halo, i.e., the number of halo stars is simply twice the number of counter-rotating stars. This provides an estimate for the level of halo contamination in each bin, which allows us to make corrections when calculating our kinematic properties. Some authors have argued that the local stellar halo is rotating (e.g. Carollo et al. 2007), although this is in conflict with later studies (e.g. Ivezić et al. 2008; Smith et al. 2009a) which show that it is consistent with little or zero rotation.⁴

The resulting halo fractions are listed in Table 1, from which it can be seen that in general halo contamination is small. For all except the most metal-poor bins, the fraction of halo stars is less than 5 per cent and usually no more than 1 per cent. However, the lowest range in metallicity, which contains the largest fraction of halo stars, may be susceptible to halo contamination if the subtraction is imperfect. As expected, the number of halo stars increases as we move further from the plane. It should be noted that due to the spectroscopic selection function of the SDSS survey, we do not expect these values to be representative of a volume-limited sample.

3.2. Velocity Dispersions

We use a maximum likelihood method to fit each v_R and v_z distribution as the sum of the disk and halo distributions, which are modelled using Gaussians. Although these velocity distributions are not expected to be exactly Gaussian, a maximum likelihood technique provides more robust results than simply calculating the sample variance. The relative normalization is the halo fraction as calculated in Section 3.1, and the velocity dispersions for the halo are fixed at $\sigma_R = 138.2 \text{ km s}^{-1}$ and $\sigma_z = 89.3 \text{ km s}^{-1}$ (Smith et al. 2009a) with no bulk motion. Note that in the calculation of the maximum likelihood, we take into account the errors on the individual velocities, as estimated in Section 2.

We also use a maximum likelihood method to fit the v_ϕ distribution. However, it is well known that the distribution of v_ϕ for the disk is highly skewed and non-Gaussian (e.g. Strömberg 1927; Evans & Collett 1993; Cuddeford & Binney 1994; Binney & Merrifield 1998). Therefore, we require an asymmetric model to fit this distribution and adopt the functional form described in equation (15) of Cuddeford & Binney (1994), namely the distribution

⁴ Our results have been found using the assumption that the halo is not rotating. However, for comparison purposes, we investigate the effect that a rotating halo would have on our results (following the results of Carollo et al. 2007). This has a tendency to slightly increase our estimated halo fractions, but the effect on our results are small and our conclusions remain unchanged. Note also that the halo rotation is entirely degenerate with the assumed value for the local standard of rest, which we have taken to be 220 km s^{-1} (see, for example, Deason, Belokurov & Evans 2011).

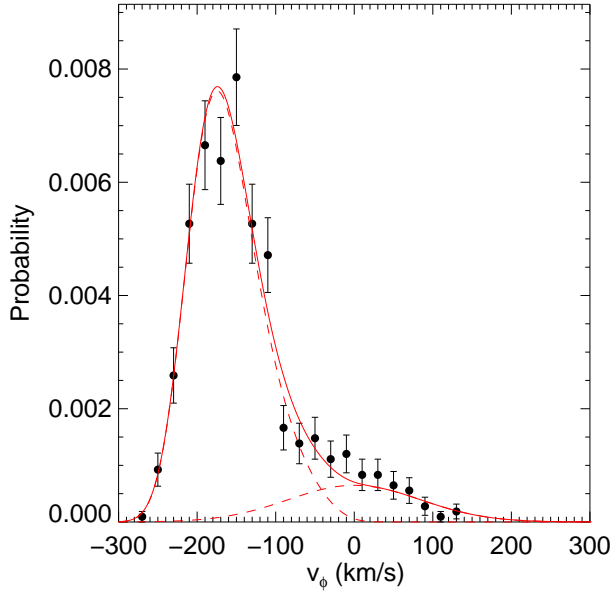


FIG. 1.— An example of one of the fits to our distribution of v_ϕ , using the method described in Section 3.2. This figure shows the most metal-poor bin for stars at heights of around 1 kpc below the plane (corresponding to the 10th column of Table 1).

function for v_ϕ is given by,

$$f(v_\phi) \propto \exp\left(-\frac{v_R^2}{2\sigma_d^2} e^{y v_\phi / v_c}\right) \times \exp\left\{\frac{1}{2\sigma_d^2} \left[v_\phi^2 - v_c^2 + 2v_c^2 \ln\left(\frac{v_c}{v_\phi}\right)\right] e^{y v_\phi / v_c}\right\}, \quad (1)$$

where $y = 8 \text{ kpc}/R_d$.

This results in a prediction for the distribution of v_ϕ as a function of two free parameters (v_c and R_d). To remove the dependence on v_R in this equation we marginalise over the known distribution of v_R for the disk stars in a given bin, as estimated using the method described in the previous paragraph. We use the observed v_ϕ distribution to calculate the likelihood distributions for v_c and R_d . Given these likelihood distributions, we can then take the moments of equation (1) to estimate the mean and dispersion of v_ϕ (and their uncertainties). Note that we do not infer any physical meaning from our determination of v_c and R_d – we are simply using the functional form given in equation (1) as a convenient description. We find that in most cases this provides a reasonable fit to the v_ϕ distribution and should provide a more robust estimate than a sample variance. As before, we include a non-rotating Gaussian halo component in the fit, with the dispersion fixed at 82.4 km s^{-1} (Smith et al. 2009a). An example fit is shown in Figure 1.

3.3. Covariance

The covariance of the disk is calculated for each bin from the observed covariance using the relation

$$\sigma_{ij} = \langle v_i v_j \rangle_d = \frac{\langle v_i v_j \rangle_{dh} - f_h \langle v_i v_j \rangle_h}{1 - f_h}, \quad (2)$$

where f_h is the calculated value of the halo fraction, the subscripts ‘dh’ and ‘h’ refer to the combined disk plus

halo sample and the halo respectively, and the subscripts i and j denote the coordinates R , ϕ or z . In practice, we only analyze two cases, namely the tilt in the ellipsoid (i.e. $\langle v_R v_z \rangle$) and the vertex deviation (i.e. $\langle v_R v_\phi \rangle$). Note that when we calculate $\langle v_i v_j \rangle_{dh}$ we subtract the mean, so this should explicitly be written $\langle (v_i - \langle v_i \rangle)(v_j - \langle v_j \rangle) \rangle_{dh}$.

For the halo stars, we assume that the ellipsoid is aligned in spherical polar coordinates, or in other words, it is pointing towards the Galactic centre. This has been shown to be a good approximation by Smith et al. (2009b), who measured the offset to be no more than a few degrees for a sample of halo subdwarfs.

Unlike the previous quantities, which were calculated using the maximum likelihood method, it is not possible to incorporate the errors on each individual velocity into this covariance calculation. To account for the effect of the errors, we instead adopt the following correction. For a particular star, the measured velocity, v_i^m , is equal to its true value, v_i^t , plus some displacement, Δv_i , which is due to the error in the measurement, i.e. $v_i^m = v_i^t + \Delta v_i$. Given this, the true value of $v_i v_j$ for this star is related to the measured value by

$$(v_i v_j)^t = (v_i v_j)^m - v_i^m \Delta v_j - v_j^m \Delta v_i - \Delta v_i \Delta v_j. \quad (3)$$

If we obtain the covariance by averaging this measurement over all stars in the bin, then clearly $\langle v_i^m \Delta v_j \rangle$ and $\langle v_j^m \Delta v_i \rangle$ should vanish. Therefore, we need only to correct $\langle v_i v_j \rangle$ by subtracting the term $\langle \Delta v_i \Delta v_j \rangle$, which we can approximate using our Monte Carlo estimates for the uncertainties (see Section 2).

This correction is important at high z , where the uncertainties on the tangential velocity are much higher. We find that this is especially problematic for the analysis of σ_{Rz} , since there are a larger fraction of stars at large R (and hence with large positive values of $\langle \Delta v_R \Delta v_z \rangle$). In order to make our measurement more robust for σ_{Rz} , we reduce the radial range and exclude any stars outside of $7.5 \text{ kpc} < R < 8.5 \text{ kpc}$. This reduces the number of stars in each bin (especially for large z , where some bins now have around 300 stars) and subsequently increases our statistical uncertainties, but we believe this is an acceptable trade-off in order to reduce systematic errors.

Once we have calculated the covariances, we use the following formula to obtain the angles

$$\theta_{ij} = 0.5 \tan^{-1} \left(\frac{2\sigma_{ij}}{\sigma_i^2 - \sigma_j^2} \right). \quad (4)$$

For both the covariances and the corresponding angles, we determine the uncertainties using a bootstrap technique. We take 10,000 resamples from the observed distribution of v_i and v_j , with repetition, and calculate $\langle v_i v_j \rangle$ for each resample, taking the dispersion of the resulting distribution as an estimate of the uncertainty in $\langle v_i v_j \rangle$. The procedure is similar for θ_{ij} , although in this case we also incorporate the uncertainties in the dispersion, as calculated according to Section 3.2.

4. KINEMATICS OF THE DISK

All of the analysis in this section refers to properties of the disk population, recovered by subtraction of the halo contaminants. The values of the kinematic properties are summarized in Table 1.

TABLE 2
BEST-FIT LINEAR RELATIONS FOR THE VELOCITY DISPERSIONS.

Component	[Fe/H] (dex)	$\sigma(z=0)$ (km s^{-1})	$\partial\sigma/\partial z$ ($\text{km s}^{-1} \text{ kpc}^{-1}$)
σ_R	(-0.5, +0.2)	32.2	11.3
σ_R	(-0.8, -0.5)	35.3	12.8
σ_R	(-1.5, -0.8)	47.6	7.9
σ_ϕ	(-0.5, +0.2)	22.4	8.7
σ_ϕ	(-0.8, -0.5)	25.9	13.0
σ_ϕ	(-1.5, -0.8)	32.1	13.3
σ_z	(-0.5, +0.2)	17.4	9.3
σ_z	(-0.8, -0.5)	28.9	7.0
σ_z	(-1.5, -0.8)	38.2	5.4

4.1. Velocity Dispersions

We plot the velocity dispersions as a function of z and metallicity in Fig. 2. The error bars represent the extent of the 1- σ confidence interval determined by the maximum likelihood fitting. We find that populations of lower metallicity are hotter than their more metal-rich counterparts. This is a natural consequence of the correlation between age and metallicity - the metal-poor stars are typically older and hence have had more time to be affected by the heating mechanisms which cause disk stars to evolve away from the cold, circular orbits on which they were born. We also see that the velocity dispersions increase on moving away from the Galactic plane, though there are hints that the velocity dispersion σ_z may be saturating at high z . Table 2 reports the best-fit linear relations for these dispersions. Most are reasonably well approximated by a linear fit, with the arguable exception of σ_z which appears to saturate at large $|z|$.

Such features have been found in a number of previous studies. For example, Binney & Merrifield (1998, section 10.4.1) use a sample of nearby stars (Strömgren 1987) to show how the velocity dispersions vary as a function of metallicity. These values are consistent with what we might expect if we extrapolate our results back to $z = 0$. More recent analyses have been done in the immediate solar neighborhood using data from the Hipparcos satellite (e.g. Dehnen & Binney 1998a) or the Geneva-Copenhagen survey (e.g. Nordström et al. 2004), all of which are consistent with an extrapolation of our results. Analyses out of the plane have been limited due to the difficulty of obtaining reliable six-dimensional phase-space information for distant stars, but there are a number of such studies. For example, Spagna et al. (2010) investigate trends in σ_ϕ with both [Fe/H] and z and Soubiran et al. (2008) attempt to address the relation between age and velocity dispersion for clump giants towards the North Galactic Pole. The variation in σ_z with z , which is important as it can be used to trace the vertical potential of the disk (see Section 5), has received a significant amount of attention, most notably in Kuijken & Gilmore (1989) and more recently in Bond et al. (2010). Binney (2010) has produced a distribution function model which is able to predict the variation in σ_z and we find that our results are in good agreement with this, although a detailed comparison is difficult due to the fact that we probe three separate metallicity ranges and do not provide an overall profile.

Can we use the information contained in Fig. 2 to ad-

dress the nature of the heating mechanisms? There are a variety of different phenomena that could act to heat the disk, from secular processes such as scattering due to spiral arms or molecular clouds (see section 8.4 of Binney & Tremaine 2008 and references therein), or external processes such as accretion of satellites onto the Milky Way (e.g. Villalobos & Helmi 2008). It has been postulated that the eccentricity structure of the thick disk can be used to discriminate formation models (Sales et al. 2009; Di Matteo et al. 2011). Recently a number of papers have investigated this using observational data (Dierickx et al. 2010; Wilson et al. 2011). One of these works (Dierickx et al. 2010) looks at the eccentricity distribution as a function of both height and metallicity and finds clear gradients, with the lower metallicity populations and those further from the plane are on more eccentric orbits; this matches the picture given by our data if we take σ_R as a proxy for eccentricity.

These heating mechanisms may also result in differences in the predicted anisotropies for the velocity ellipsoid. In order to investigate this, we plot the ratios of the dispersions in Fig. 3. The solar neighborhood estimate for $\sigma_z/\sigma_R \approx 0.5$ (Wielen 1977; Dehnen & Binney 1998a; Nordström et al. 2004), is consistent with our metal-rich sample. We detect a general increase in this ratio for lower metallicities, seemingly due to the fact that σ_z has a stronger dependence on metallicity than σ_R , i.e. σ_z increases faster than σ_R as metallicity decreases, resulting in larger values for σ_z/σ_R . There is very little evidence for a gradient in z , and only a mild hint that the ratio is increasing with z .

Theoretical studies of the expectations for σ_z/σ_R predict a range of values less than 1. Classical work by Jenkins (1992) looked at heating by molecular clouds and spirals, and predicts ratios varying from 0.4 to 0.8 depending on the strength of the spirals, while a study into the effects of heating by molecular clouds alone predicts a value of around 0.6 (Ida, Kokuno & Makino 1983). Our results are in keeping with these predictions. In general, it is thought that scattering off spiral arms is more efficient at heating in the plane, while molecular clouds are more efficient at heating perpendicular to the plane. In this picture one might interpret the gradient with metallicity to claim that more metal-poor stars (i.e. older and hotter populations) are less affected by heating from spiral arms. Whatever the reason, it is clear that the metal-poor stars are significantly more isotropic in their kinematics than their metal-rich counterparts.

Simulations of satellite accretion also predict similar ratios for this ratio. For example, Villalobos & Helmi (2008) and Villalobos, Kazantzidis & Helmi (2010) simulate the formation of the thick disk through heating via accretion, and also predict a wide range of values for σ_z/σ_R (from ~ 0.4 to 0.9) depending on the parameters chosen to set-up their simulations (e.g. satellite mass ratio, orbital inclination, etc). Therefore, it seems that theoretical expectations cover a range of values, meaning that our data does not simply exclude a particular mechanism. However, now that the velocity dispersions can be found as a function of height and metallicity, it should help to constrain future theoretical models by excluding specific ranges of parameter space. Very few models are able to predict the detailed behavior of these ratios and how they vary as a function of metallicity or z , which will

clearly become more important as models are refined.

In recent years, there have been a number of attempts to measure σ_z/σ_R for external galaxies. Even though such determinations are model dependent and based on a number of assumptions, it is interesting to compare these to the Milky Way. Gerssen, Kuijken & Merrifield (1997, 2000) looked at the large spiral galaxies NGC 488 and NGC 2985, finding ratios of 0.7 and 0.8, respectively, i.e. slightly larger than the classical Milky Way determination of 0.5. Similar results were also found by van der Kruit & de Grijs (1999) in their study of a compilation of around 40 edge-on disk galaxies, with typical values ranging from 0.5 to 0.7 and with no evidence for a correlation between these values and the galaxy size or morphological classification (from Sb to Sd). Although larger than the Milky Way value, these ratios for external galaxies are consistent with our data; as can be seen in Fig. 3, as one departs from the plane and moves to lower metallicities values of $\sigma_z/\sigma_R = 0.5 - 0.8$ are typical.

The ratio σ_ϕ/σ_R also provides information about heating processes, though this has received less attention in the literature. The value in the solar neighborhood is of interest as it is related to the slope of the rotation curve of the Galaxy. The ratio σ_ϕ^2/σ_R^2 , which is sometimes referred to as Oort's ratio, is predicted to lie around 0.5 to 0.6 depending on the shape of the rotation curve (Kuijken & Tremaine 1991; see also Evans & Collett 1993). An extrapolation back to the solar neighborhood gives a good agreement with this expectation, similar to the results of Dehnen & Binney (1998a). In Fig. 3, we can see that the gradient with metallicity is less prominent than for σ_z/σ_R , although it also looks like in general the metal-poor stars are more isotropic. As with σ_z/σ_R , there is only weak indication that stars further from the plane exhibit larger ratios; in particular it is worth noting that the distribution for the metal-rich stars is essentially flat.

4.2. The Rotation Lag

It has long been understood that there is a correlation between the speed at which a population of stars rotates around the Galactic centre and the velocity dispersion of this population. This is known as the asymmetric drift (e.g. section 10.3.1 of Binney & Merrifield 1998) and is clearly evident in the solar neighborhood (e.g. Dehnen & Binney 1998a). The behavior out of the plane is more controversial. Whilst the existence of a correlation between lag and height from the plane has been observed in numerous studies (e.g. Wyse & Gilmore 1986; Majewski 1992; Chiba & Beers 2000; Girard et al. 2006), the situation regarding trends with metallicity are less clear. In recent years there have been at least two papers with seemingly irreconcilable views; Spagna et al. (2010) claim to have detected a large gradient, at a level of around 40 to 50 $\text{km s}^{-1} \text{dex}^{-1}$, while Ivezić et al. (2008) claim an essentially flat relation with no detectable gradient.

Our findings are presented in Fig. 4, where it is immediately evident that there are clear trends in the lag. The hotter populations (i.e. those with lower metallicity or those further from the plane) exhibit significantly more lag than their colder counterparts, with the metal-poor stars rotating more than 80 km s^{-1} slower than the LSR at 2 kpc from the plane. The gradient of the lag with respect to $|z|$ varies from around 15 to 40 $\text{km s}^{-1} \text{kpc}^{-1}$, de-

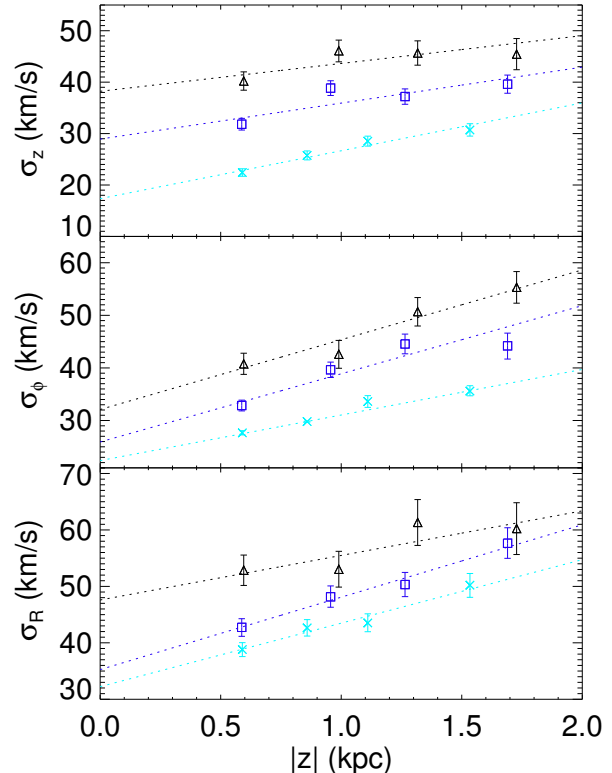


FIG. 2.— Dispersions σ_R , σ_z and σ_ϕ as functions of z and metallicity. The triangles, squares and crosses correspond to metallicity ranges $-1.5 \leq [\text{Fe}/\text{H}] \leq -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $-0.5 \leq [\text{Fe}/\text{H}] \leq 0.5$, respectively.

pending on metallicity. This is in agreement with many of the literature values which typically find values around 30 $\text{km s}^{-1} \text{kpc}^{-1}$ (e.g. Girard et al. 2006). It is curious to note that for the more metal-rich stars there appears to be a saturation in the level of the lag as one moves from the plane; from Fig. 4 it can be seen that while the metal-poor stars exhibit an almost linear trend, the gradient for the metal-rich and intermediate-metallicity populations becomes shallower for $z \gtrsim 1$ kpc.

Despite the strong theoretical grounds for expecting the lag to be larger for hotter (and hence more metal-poor) populations, as is seen in our data, some authors disagree with this claim. In particular, Ivezić et al. (2008) have argued that such a trend with metallicity is not observed in a sample of SDSS data. Their viewpoint is seemingly lent weight by the findings of Loebman et al. (2010), who analyzed an N-body+SPH disk galaxy simulation and found no clear trend when plotting rotation velocity versus metallicity (see fig. 10). However, this does not imply that such a trend is entirely absent from their simulations; their fig. 9 shows that young stars do indeed possess larger rotation velocities compared to the older stars and also that there is a clear age-metallicity gradient. If instead Loebman et al. (2010) slice their data in the same manner as we have (i.e. taken four bins in $|z|$ from 0.5 to 2 kpc and three bins in metallicity), then they see behavior which is qualitatively identical to our Fig. 4 with a clear gradient in metallicity (V. Debattista, private communication).

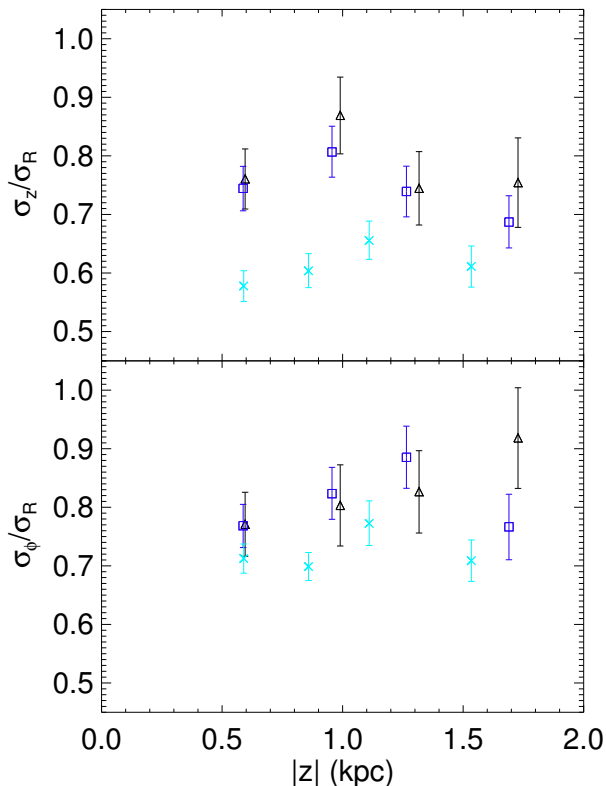


FIG. 3.— The ratio of σ_z to σ_R and σ_ϕ to σ_R as a function of z . The triangles, squares and crosses correspond to metallicity ranges $-1.5 \leq [\text{Fe}/\text{H}] < -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] < -0.5$ and $-0.5 \leq [\text{Fe}/\text{H}] \leq 0.5$, respectively.

In the solar neighborhood, Dehnen & Binney (1998a) found that there is a linear relation between lag and the square of the radial velocity dispersion, which is in good agreement with theory (e.g. section 4.8.2a of Binney & Tremaine 2008). However, it is less clear what is expected as we depart from the plane. We show the observational result for our data in Fig. 5. As in the solar neighborhood, there appears to be a relatively tight correlation with σ_R^2 that is independent of metallicity. However, the gradient is much steeper than that of the solar neighborhood, where it is found that $\langle v_\phi \rangle - v_{\text{LSR}} = \langle v_R^2 \rangle / (80 \pm 5 \text{ km s}^{-1})$. The relation is reasonably well-fit by an quadratic relation, as shown in Fig. 5, where we find,

$$\langle v_\phi \rangle - v_{\text{LSR}} = 0.0149\sigma_R^2 + 1.21 \times 10^{-6}\sigma_R^4. \quad (5)$$

Note that our analysis has been carried out using the Dehnen & Binney (1998a) value for the Sun's motion with respect to the local standard of rest. This has recently been revised by Binney (2010), with the velocity in the direction of rotation being increased from 5.2 km s^{-1} to 11 km s^{-1} . If we adopt this newer value, then it implies that the lag which we measure will be too large by around $\sim 5 \text{ km s}^{-1}$. However, since this offset should be applied uniformly to all of our sample, the gradients in lag with metallicity and height from the plane, which we can see from Fig. 4, will be unaffected.

4.3. Radial and Vertical Bulk Motions

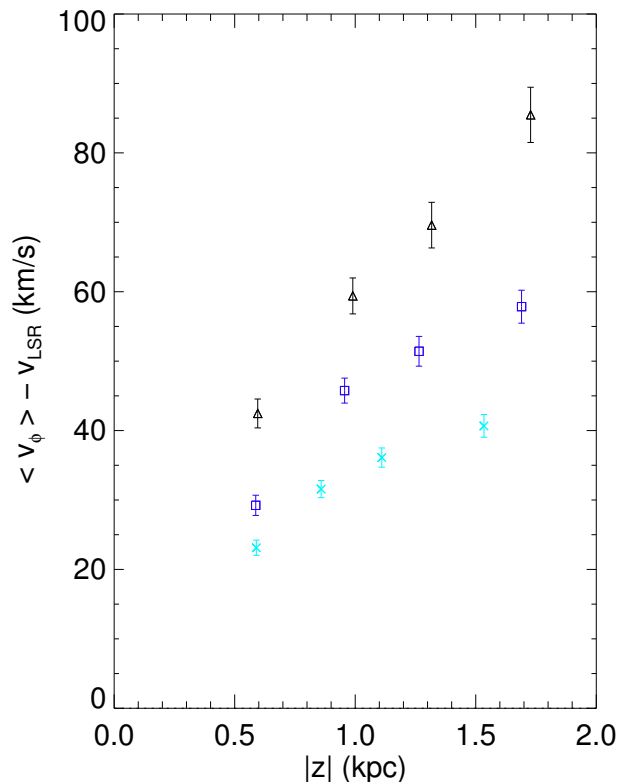


FIG. 4.— The rotational lag, plotted against z . Velocities are relative to the Local Standard of Rest, assumed to be as determined by Dehnen & Binney (1998a), which means that in this plot the Sun would lie at $(0, -5 \text{ km s}^{-1})$. The triangles, squares and crosses correspond to metallicity ranges $-1.5 \leq [\text{Fe}/\text{H}] < -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] < -0.5$ and $-0.5 \leq [\text{Fe}/\text{H}] \leq 0.5$, respectively.

The bulk motion for the other two components, v_z and v_R are presented in Fig. 6 and Table 1. Curiously $\langle v_z \rangle$, whilst appearing consistent with zero for the metal-poor stars, seems to deviate from zero on moving away from the plane for the more metal-rich samples. Although tentative, this is an intriguing result and, if confirmed by further observations, could have important implications.

In the lower panel of Fig. 6, we see that the radial component also seems to exhibit a net motion, with the sample moving outwards from the Galactic centre. This has also been seen in data from the RAVE survey (Siebert et al. 2011), using a sample of stars with $|z| < 1 \text{ kpc}$. Here, we seem to be detecting similar behavior out to $|z| \sim 2 \text{ kpc}$, though there are no clear trends with either $|z|$ or metallicity. This is arguably surprising, since Siebert et al. (2011) suggested that such behavior should be due to non-axisymmetric components in the Galaxy, such as the bar or spiral arms, which one might expect to have a greater effect on stars in the plane. The other potential source of non-axisymmetry postulated by Siebert et al. (2011) was that of ellipticity in the outer dark matter halo, which we might expect to lead to a lack of a gradient with $|z|$.

However, there are difficulties associated with measuring such quantities and various systematic biases could affect these results, for example mistaken assumptions regarding the Local Standard of Rest or the Sun's pecu-

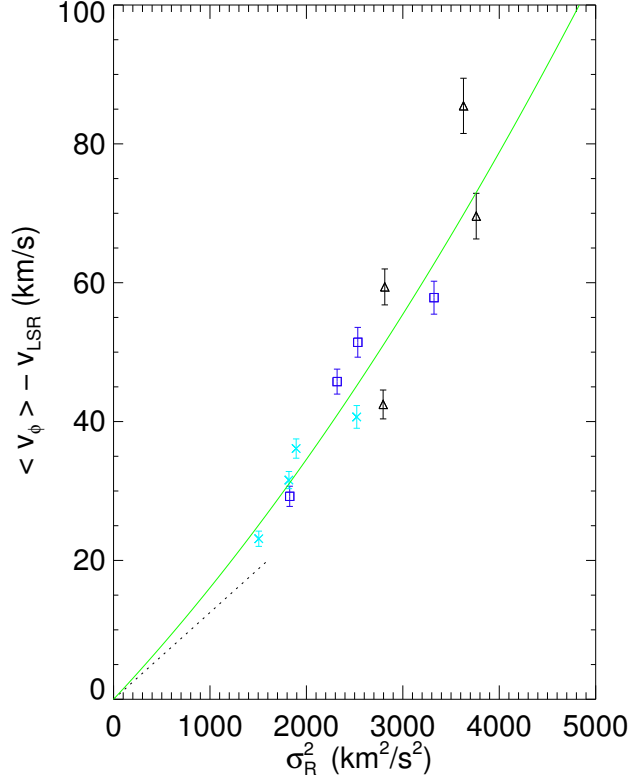


FIG. 5.— Rotational lag as a function of radial velocity dispersion. The triangles, squares and crosses correspond to metallicity ranges $-1.5 \leq [\text{Fe}/\text{H}] \leq -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $-0.5 \leq [\text{Fe}/\text{H}] \leq 0.5$, respectively. The dotted line corresponds to the solar-neighborhood relation from Dehnen & Binney (1998a). The solid line denotes an empirical fit with the lag equal to $0.0149\sigma_R^2 + 1.21 \times 10^{-6}\sigma_R^4$.

liar motion.

4.4. Tilt of the Velocity Ellipsoid

The covariance between v_ϕ and v_z , which is often referred to as the tilt term, is an important quantity, which we can attempt to determine with our data. Little is known about how this behaves when one moves away from the Galactic plane. At $z = 0$, we expect this to vanish due to symmetry arguments, but the behavior out of the plane is harder to measure as it requires distances to be determined to high accuracy. Despite these difficulties, the orientation of the ellipsoid is of importance for understanding the shape of the potential (c.f. Smith et al. 2009b; Binney & McMillan 2011; Evans 2011). Theoretical predictions span a range of values lying anywhere between velocity ellipsoids parallel to the Galactic plane to ones which point towards the Galactic centre (Binney & Spergel 1983; Kuijken & Gilmore 1989). The consensus (e.g. Binney & Tremaine 2008) seems to be that an alignment midway between cylindrical polar and spherical polar is most reasonable.

One of the recent determinations of the tilt was by Siebert et al. (2008). They used red clump giants from the RAVE survey to measure a tilt angle of $7^\circ.3 \pm 1^\circ.8$ at $z \sim 1$ kpc below the plane, with the orientation inclined towards the Galactic centre. Their sample of stars covers a large range in z (from 0.5 to 1.5 kpc), which

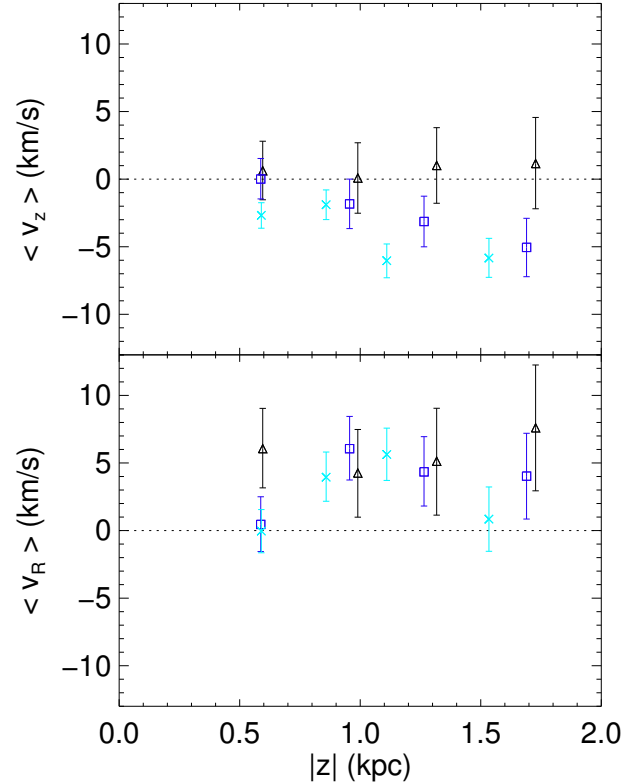


FIG. 6.— Mean velocities as a function of height from the plane. The triangles, squares and crosses correspond to disk stars with metallicities $-1.5 \leq [\text{Fe}/\text{H}] \leq -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $[\text{Fe}/\text{H}] \geq -0.5$, respectively.

complicates the interpretation of their result, but it is clear that this value is close to what one would expect for an ellipsoid aligned with the centre of the Galaxy (i.e. $\tan^{-1}1/8 = 7^\circ.1$). This is an unexpected result as it lies at the extremum of the predicted values, but it appears to be robust as it has been supported by subsequent studies (Casetti-Dinescu et al. 2011).

Our results are given in Table 1. We have omitted the values for the metal-poor stars due to problems in accurately measuring their covariances. This is because it is very difficult to reliably correct for the halo contamination - for the metal-poor sample the covariance measurements are dominated by the contribution from the halo stars (since σ_{Rz} for the halo is much larger than for the disk) and so reliably extracting the disk covariance is practically impossible. We include the medium-metallicity sample in our analysis as the halo contamination is smaller and hence our determination more robust.

The covariances are plotted in Fig. 7. There appears to be a weak trend, with the magnitude of σ_{Rz} increasing slightly as we move away from the plane. If we now convert the covariances into an angles using equation (4), we obtain the results shown in the lower panel of Fig. 7. The dotted line corresponds to what we would expect for a velocity ellipsoid aligned in spherical polar coordinates, and so we would expect our data to lie between this dotted line and the $\alpha_{Rz} = 0$ axis. Although there are large uncertainties, it appears that the metal-rich and

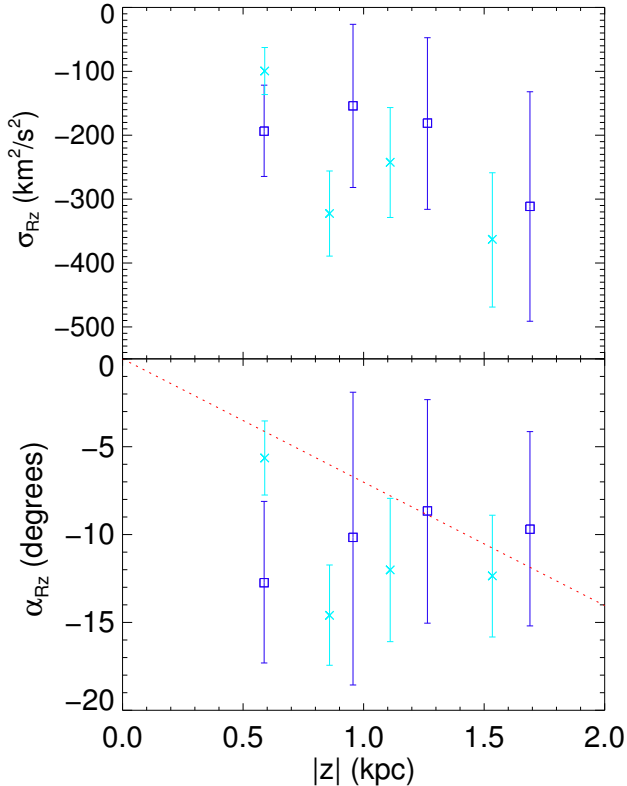


FIG. 7.— The variation of σ_{Rz} and the corresponding angle α_{Rz} , often referred to as the tilt, as a function of height from the plane. The dashed line is the assumed halo tilt (i.e. aligned in spherical polars). The squares and crosses correspond to disk stars with metallicities $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $[\text{Fe}/\text{H}] \geq -0.5$, respectively.

medium-metallicity stars are in general consistent with the dotted line (and hence consistent with the Siebert et al. 2008 result). A couple of points are more than one-sigma away from this line, but we believe this is probably an artifact; it is very difficult to explain such behavior for a disk in equilibrium, which implies that either our disk is not in equilibrium (due to accretion remnants affecting the distribution, or due to transient effects from resonances associated with spiral arms or the bar) or our data are suffering from observational biases.

To conclude, although this is a difficult measurement with large uncertainties, it does appear that the tilt angle for the disk is surprisingly large, reinforcing the findings of Siebert et al. (2008).

4.5. The Vertex Deviation

The orientation of the velocity ellipsoid in the (v_R, v_ϕ) -plane, often referred to as the vertex deviation, is also of interest. In the immediate solar neighborhood, this has been found to be around 10° to 20° (Dehnen & Binney 1998a), with corresponding covariance $\sigma_{R\phi} \sim 100 \text{ km}^2\text{s}^{-2}$. For an axisymmetric system this term should vanish and the fact that it is non-zero is usually attributed to resonances from either the Galactic bar or spiral arms (e.g. Dehnen 1999; Quillen 2003; De Simone et al. 2004; Minchev et al. 2010).

With our data we are able to probe the vertex devi-

ation out of the plane. Unlike the tilt angle discussed in the previous section, the vertex deviation is somewhat easier to measure. Since our data are located towards the South Galactic cap, velocities in the (v_R, v_ϕ) -plane are mostly derived from the proper-motion data and, as a consequence, correlated uncertainties are less of a problem. The halo correction is also easier to handle, although it still makes it difficult to robustly measure $\sigma_{R\phi}$ for the metal-poor sample. Despite the fact that the halo $\sigma_{R\phi} = 0$, the mean v_ϕ is significantly offset from that of the disk and the dispersions are large. Therefore, unless the halo velocity distributions are well sampled (which they are not), there are likely to be large statistical fluctuations due to the influence of the halo contamination between different bins, even though the mean correction will be zero.

Our results are presented in Fig. 8 and Table 1. As expected, there are large fluctuations in the measurement of $\sigma_{R\phi}$ for the most metal-poor stars. For the rest, there is a clear positive signal, which is consistent with the value found by Dehnen & Binney (1998a). If we concentrate on the most metal-rich stars, which have much smaller errors than the more metal-poor stars, the distribution appears to be flat (assuming $\sigma_{R\phi} = 100 \text{ km}^2\text{s}^{-2}$ at $z=0$) with a small increase for bin furthest from the plane. A similar rise is also seen for the medium-metallicity sample. Instead of this weak positive gradient, we might naively expect that $\sigma_{R\phi}$ should *decrease* with increasing height from the plane, since this offset is thought to be due to features which are stronger in the plane (namely the bar and spiral arms). However, this is only a very weak detection and so further study is required before any definitive statements can be made.

5. AN APPLICATION: THE GALACTIC POTENTIAL

There are many applications of the data we have provided. We now show one such example where we use the vertical dispersions to compute the Galactic potential close to the disk.

For an isothermal disk it can be shown that,

$$\frac{1}{\nu} \frac{\partial(\nu \sigma_z^2)}{\partial z} = -\frac{\partial \Phi}{\partial z} \quad (6)$$

To obtain this relation we have neglected the contribution of the tilt term (σ_{Rz}); even if the tilt is pointing towards the Galactic centre (as suggested by Fig. 7), then the contribution of these neglected terms to equation (6) will be at a level of around five per cent (see equation 4.271 of Binney & Tremaine 2008). From equation (6) it is trivial to show that,

$$\nu(z) = \nu(0) \exp \left[\frac{\Phi(0) - \Phi(z)}{\sigma_z^2} \right]. \quad (7)$$

However, it is clear that the disk is not isothermal. If we generalize equation (6) to a population with dispersion $\sigma_z(z)$ then we obtain the following formula,

$$\frac{\nu(z)}{\nu(0)} = \frac{\sigma_z^2(0)}{\sigma_z^2(z)} \exp \left[\int dz \frac{1}{\sigma_z^2} \frac{\partial \Phi}{\partial z} \right]. \quad (8)$$

By taking a potential of the form,

$$\Delta \Phi = \Phi(z) - \Phi(0) = a|z| + b|z|^2 + \dots \quad (9)$$

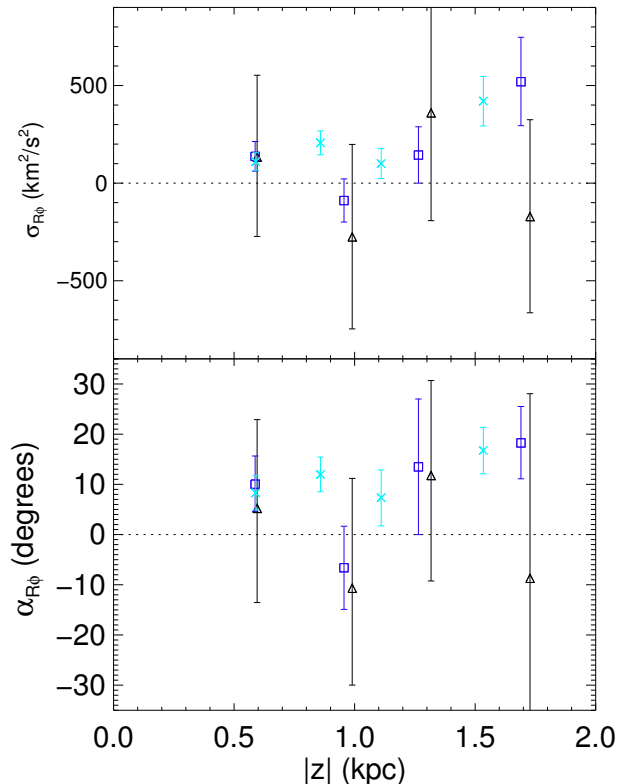


FIG. 8.— The variation of $\sigma_{R\phi}$ and the corresponding angle $\alpha_{R\phi}$, often referred to as the vertex deviation, as a function of height from the plane. The triangles, squares and crosses correspond to disk stars with metallicities $-1.5 \leq [\text{Fe}/\text{H}] \leq -0.8$, $-0.8 \leq [\text{Fe}/\text{H}] \leq -0.5$ and $[\text{Fe}/\text{H}] \geq -0.5$, respectively.

and retaining only the first two terms, we obtain the following relation,

$$\nu(z) \sigma_z^2(z) = \nu(0) \sigma_z^2(0) + \int_0^z dz' \nu(z') [a + 2bz']. \quad (10)$$

This equation allows us to constrain the potential of the disk using our measured dispersions, provided we know the form of the density distribution for our tracer populations, i.e. our three metallicity ranges. Unfortunately this proviso is not an easy one to overcome. The complicated nature of the SDSS spectroscopic selection function prohibits us from using our current spectroscopic dataset to determine the density distribution. Therefore we resort to photometric number counts, using the overall density distribution of Jurić et al (2008), convolved with the metallicity distribution function of Ivezić et al. (2008, as revised in the appendix of Bond et al. 2010). We cannot use these density distributions directly as they provide unrealistic predictions for the behavior at small z where, for the intermediate- and low-metallicity ranges, the resulting density actually rises as z increases (see the lower panel of Fig. 9). To remedy this we assume that each of our three components follow a sech to the power 0.4 profile (Banerjee & Jog 2007) and fit these functions to the Jurić/Ivezić profile over the range $0.5 < z/\text{kpc} < 2$. For the range $0 < z/\text{kpc} < 0.5$, where the individual profiles are unreliable, we require the sum of our three density distributions to match the

Jurić total density distribution. We therefore include the scale-heights and normalizations of the three components as free parameters in our fit. Since the Jurić disk profile consists of a thin- and thick-component, we similarly assume each of our metallicity populations consists of two components, keeping the scale-height ratio and the normalization of the thin- and thick-component fixed at the values adopted by Jurić ($z_{h,\text{thick}}/z_{h,\text{thin}} = 3$ and $\nu_{\text{thick}}(0)/\nu_{\text{thin}}(0) = 0.13$).

This means that we have a total of 11 free parameters: two to describe the potential plus three parameters per metallicity range ($\sigma_z(0)$ and a normalization and scale-height for the thin-component of the density profile). We then fit simultaneously the Jurić/Ivezić density profiles and the dispersion profiles (via equation 10) by using a standard χ^2 technique. We assume that the errors on the density profiles scale with $\sqrt{\nu}$.

Since our data do not constrain the dispersion profile for $z \lesssim 0.5$ kpc, we add an additional metal-rich data-point for the immediate solar-neighborhood using data from the Geneva-Copenhagen survey (Nordström et al. 2004; Holmberg, Nordström & Andersen 2009). We take the 252 stars with $[\text{Fe}/\text{H}] > -0.5$ dex, parallax error less than 13 per cent and distances less than 100 pc and find that the $\sigma_z = 15.1 \pm 0.7 \text{ km s}^{-1}$.

As we are simultaneously fitting both the density and the dispersion profiles, it is in some sense arbitrary how we weight these two components of the fit. We have chosen to construct our fit so that the overall χ^2 is approximately equal for each of the two components, which implies that our fit is not dominated by either the density or dispersion profiles.

The resulting best fit is shown in Fig. 9 with parameters given in Table 3. Although the Jurić/Ivezić relations are not necessarily well represented by our sech profiles, note that the total density distribution from our three components provides a good match to the overall Jurić et al (2008) distribution. However, it should be noted that the fit relies strongly on our assumed Jurić/Ivezić density profiles, which are subject to uncertainties that may bias our result. For example, one clear failing of our model is that the solar-neighborhood normalizations do not match observations. In particular $\nu(0)$ for the metal-rich sample is only around twice that of the intermediate-metallicity sample, whereas we know that in the solar-neighborhood stars with $[\text{Fe}/\text{H}] > -0.5$ make up around 95 per cent (Holmberg, Nordström & Andersen 2009). As a consequence our resulting potential should not be over-interpreted.

Given these caveats, it is reassuring to see that our potential (shown in the top panel of Fig. 9) is in good agreement with existing models. Most notably our simple model is in exceptional agreement with the models of Dehnen & Binney (1998b, hereafter DB), especially their Model 1. Although there is a slight discrepancy⁵ at small z ($z \lesssim 300$ pc), in general our potential matches these DB models to within 15 per cent out to 4 kpc. We have chosen to compare our potential to two DB models

⁵ This discrepancy is most-likely due to the fact that in our simple model for the potential (equation 9) one component of the mass distribution is an unphysical infinite razor-thin sheet. A better match at small z could be found if one distributed this mass in a more realistic manner.

(Model 1 and Model 4) because these can be considered to be two extreme cases of the models considered in their study, with Model 1 being the least halo-dominated and Model 4 being the most halo-dominated (see section 2.7 of Binney & Tremaine 2008 for a detailed comparison of these two models, where Models I and II correspond to DB Models 1 and 4). Over the range where we have kinematic data ($0.5 \lesssim z/\text{kpc} \lesssim 2$) it appears that our potential favors Model 1, namely the model where the disk dominates the circular speed at the solar neighborhood.

Once we have an estimate for $\Delta\Phi$ we can use this to probe the vertical mass distribution in the disk through Poisson's equation ($\partial^2\Phi/\partial z^2 = 4\pi G\rho$). Our simple model corresponds to an infinite razor-thin sheet with a surface mass density of $a/(2\pi G) = 32.5 \text{ M}_\odot\text{pc}^{-2}$, embedded in a uniform background with mass density $b/(2\pi G) = 0.015 \text{ M}_\odot\text{pc}^{-3}$. Note that this background mass is close to the $0.014 \text{ M}_\odot\text{pc}^{-3}$ predicted using isothermal spherical halo models (equation 4.279 of Binney & Tremaine 2008). If we assume our background mass represents the dark halo, it corresponds to a local dark matter density of 0.57 GeV cm^{-3} , which is noticeably larger than the canonical value of 0.30 GeV cm^{-3} typically assumed (e.g. Jungman et al 1996). As pointed out by various authors (e.g. Gates et al. 1995; Weber & de Boer 2010; Garbari et al. 2011), the local dark matter density is uncertain by a factor of at least 2. Our analysis adds still more weight to the argument the local halo density may be substantially underestimated by the canonical value of 0.30 GeV cm^{-3} , and this is of immediate interest to dark matter experimentalists.

Perhaps more robust than the local mass density is the surface mass density. By integrating our mass distribution we obtain a total surface mass density of $\Sigma_{1.1\text{kpc}} = 66 \text{ M}_\odot\text{pc}^{-2}$, which agrees well with the classical value of $71 \pm 6 \text{ M}_\odot\text{pc}^{-2}$ from Kuijken & Gilmore (1991). If we integrate beyond 1.1 kpc, we find $\Sigma_{2\text{kpc}} = 94 \text{ M}_\odot\text{pc}^{-2}$ and $\Sigma_{4\text{kpc}} = 155 \text{ M}_\odot\text{pc}^{-2}$.

Although we would ideally like to quote uncertainties for the above quantities, this is difficult due to the number of assumptions and approximations in the adopted prescription. In particular, this method is dependent on what we assume for the density distributions for our tracer populations (in our case, the Jurić/Ivezić profiles). As a consequence, the formal errors from the χ^2 fitting will be meaningless and we choose not to quote them. We have also experimented by adding a third term in the expansion of the potential (equation 9), but found that gave no statistical improvement to the fit and had no significant influence on the resulting potential.

6. CONCLUSIONS

We have undertaken a kinematic study of the Galactic disk using data from the SDSS equatorial stripe region towards the South Galactic cap (Stripe 82). By combining spectroscopic data from Lee et al. (2008) with proper motions from Bramich et al. (2008), we have constructed a sample of 7280 disk stars with full 3-dimensional positions and velocities, along with $[\text{Fe}/\text{H}]$. This data have allowed us to probe the kinematics of the disk, tracing means and dispersions as a function of height from the plane and $[\text{Fe}/\text{H}]$.

These data can be used to investigate the evolution of

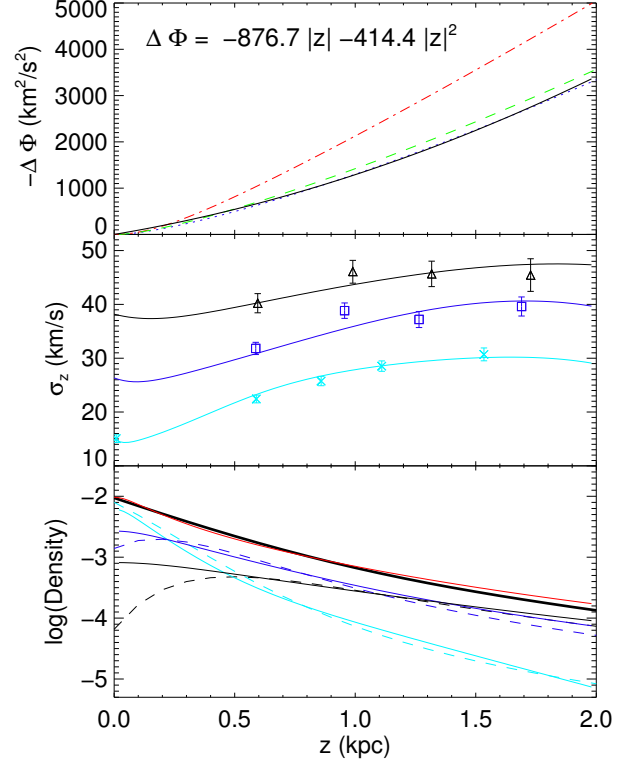


FIG. 9.— The results of the application to constrain the potential of the Galactic disk, as described in Section 5. The bottom panel shows the density distribution. In the range $0.5 < z/\text{kpc} < 2$ we fit sech profiles to the metal-rich (cyan), medium-metallicity (blue) and metal-poor (black) profiles derived from the work of Jurić/Ivezić. In the range $z/\text{kpc} < 0.5$ we fit to the total density profile (thick black line). The total of our three density components is given by the thin red line. The middle panel shows the fit to the dispersion profiles, with the data taken from Fig. 2 plus an additional metal-rich point at $z \approx 0$ from the Geneva-Copenhagen survey. The upper panel shows the potential resulting from these fits. For the purposes of comparison, included in the upper panel are models for the potential taken from literature sources, namely Dehnen & Binney (1998 – Model 1, dotted; Model 4, dashed) and Fellhauer (2006 – dot-dashed).

the disk, as stars which are born on circular orbits in the plane are heated by various mechanisms, such as the those caused by spiral arms, the bar, molecular clouds or accretion events. One avenue for addressing this is to investigate the ratio σ_z/σ_R . For the metal-rich stars we measure this to be 0.6, which is consistent with predictions for disk heating via spiral arms and molecular clouds (Jenkins 1992). However, predictions from models of satellite accretion also cover the range we observe and so it is difficult to deduce any strong conclusions. We also investigate the ratio σ_ϕ/σ_R , and find that in general stars further from the plane exhibit larger ratios, although this trend is less evident for the more metal-rich stars. It will be interesting in future for models to attempt to explain the gradients, both in z and $[\text{Fe}/\text{H}]$, which are evident in our data. In particular, it is clear that the metal-poor stars are more isotropic in their kinematics than their metal-rich counterparts.

Our data also allow us to measure the covariances and constrain the orientation of the velocity ellipsoid. We

TABLE 3

BEST-FIT PARAMETERS FROM THE POTENTIAL FITTING PROCEDURE. PARAMETERS a AND b CORRESPOND TO THE POTENTIAL GIVEN IN EQUATION (9), $\nu(0)$ IS THE SOLAR-NEIGHBORHOOD DENSITY NORMALIZATION AND $z_{h,thin}$ AND $z_{h,thick}$ ARE THE SCALE-HEIGHTS OF THE THIN AND THICK COMPONENTS OF THE DISK PROFILE, RESPECTIVELY. NOTE THAT THE SCALE-HEIGHT OF THE THICK COMPONENT IS NOT A FREE PARAMETER AND IS KEPT FIXED AT THREE TIMES THAT OF THE THIN COMPONENT (FOLLOWING JURIĆ ET AL 2008).

Parameter	[Fe/H] Range	Value
a ($\text{km}^2 \text{s}^{-2} \text{kpc}^{-1}$)	—	-876.7
b ($\text{km}^2 \text{s}^{-2} \text{kpc}^{-2}$)	—	-414.4
$\nu(0)$ ($10^{-3} M_{\odot} \text{pc}^{-3}$)	(-0.5, +0.2)	6.1
$z_{h,thin}$ (kpc)	"	0.05
$z_{h,thick}$ (kpc)	"	0.16
$\sigma_R(0)$ (km s^{-1})	"	14.9
$\nu(0)$ ($10^{-3} M_{\odot} \text{pc}^{-3}$)	(-0.8, -0.5)	2.7
$z_{h,thin}$ (kpc)	"	0.14
$z_{h,thick}$ (kpc)	"	0.42
$\sigma_R(0)$ (km s^{-1})	"	26.3
$\nu(0)$ ($10^{-3} M_{\odot} \text{pc}^{-3}$)	(-1.5, -0.8)	0.8
$z_{h,thin}$ (kpc)	"	0.26
$z_{h,thick}$ (kpc)	"	0.78
$\sigma_R(0)$ (km s^{-1})	"	38.2

found that the tilt term (α_{Rz}) is not consistent with zero, i.e. the ellipsoid is not aligned with cylindrical polar coordinates. Our results are consistent with previous studies, which have found that the vertical component of the ellipsoid is close to being aligned in spherical polar coordinates (e.g. Siebert et al. 2008). The vertex deviation ($\alpha_{R\phi}$) is found to be consistent with an extrapolation to the solar-neighborhood, with a marginal detection of an increase as one moves away from the plane. If this gradient can be confirmed it may have interesting implications, as the mechanisms that are believed to drive this term away from zero (e.g. the bar or spiral arms) should have greater influence in the plane.

In order to address the nature of disk heating and to disentangle the contributions from various mechanisms, one needs to go beyond the work presented here. The most crucial improvement will be the ability to make direct estimates for stellar ages, rather than relying on correlations with metallicity. Ages are notoriously difficult to measure robustly, even for surveys of bright nearby stars (e.g. Holmberg, Nordström & Andersen 2009). Extending such studies beyond our immediate solar-neighborhood will be a difficult task, but one that is currently showing promise (e.g. Burnett et al. 2011). One aspect that will help us in this effort is by folding in measurements of alpha-element abundances that are now being determined routinely for vast numbers of stars (e.g. Bovy et al. 2011; Lee et al. 2011; Navarro et al. 2011; Ruchti et al. 2010). In particular Bovy et al. (2011) and Lee et al. (2011) show what is possible with SDSS data when one incorporates information on alpha-element abundances.

In recent years there has been some debate regarding the rotation lag of the disk. The lag has been known about for many years, with recent determinations of around $30 \text{ km s}^{-1} \text{kpc}^{-1}$ (e.g. Girard et al. 2006). We find

gradients of around 15 to $40 \text{ km s}^{-1} \text{kpc}^{-1}$. These gradients depend on height from the plane (with the gradients in general becoming shallower as z increases) and also on metallicity. Our findings address one particular bone of contention, which is whether there exists a correlation between rotation lag and metallicity (e.g. Ivezić et al. 2008; Spagna et al. 2010). Our data clearly show that the more metal-poor (and hence hotter) stars exhibit greater lag than their metal-rich counterparts, with the metal-poor stars reaching a lag of more than 80 km s^{-1} at 2 kpc from the plane. Our findings are in good qualitative agreement with simulations, although further work needs to be done to fully exploit these observational results.

In passing, it is interesting to compare our results on the vertical gradient of the lag to that for neutral HI gas. For the Milky Way this has been found to be $15 \pm 4 \text{ km s}^{-1} \text{kpc}^{-1}$ (Marasco & Fraternali 2011), which is comparable to our findings for metal-rich stars. This measurement is arguably easier to determine for external edge-on disk galaxies, where it has been found to be $10 - 30 \text{ km s}^{-1} \text{kpc}^{-1}$ (Heald et al. 2007; Kamphuis et al. 2007; Zschaechner et al. 2011). Although the gas response to the various heating mechanisms will differ from that of the collisionless stars, the magnitude of the lags appear similar.

Another limitation of our analysis is that we are confined to the 250 square degrees of Stripe 82. Although we are able to probe gradients with height from the plane, we are unable to draw any conclusions regarding radial gradients. Furthermore, the limited sky coverage means that we are susceptible to bias should kinematic substructure be present within any of our bins. Although no such substructure is immediately evident in our data, we cannot exclude such an occurrence. By extending this to the whole SDSS footprint using the recently released 8th Data Release (Aihara et al. 2011) it will be possible to overcome some of these limitations, although the lack of radial coverage will persist as SDSS has very few fields at low Galactic latitude. Future work that will help to fill in the missing information at low latitudes includes the LAMOST telescope (e.g. Wu et al. 2011) surveys, which will in-part focus on the Galactic disk. Looking further ahead the field will undoubtedly be revolutionized by the Gaia satellite mission, which will provide precise distances and proper motions for billions of stars in our Galaxy (Perryman et al. 2001; Sumi 2009).

We have also shown an application of our data, employing the Jeans equations to provide a simple model of the Galactic potential close to the disk. We use the density model of Jurić et al (2008) and our measured values on the variation of σ_z . Our model does a good job of reproducing the kinematic data for stars in all three ranges of metallicity. The model is in excellent agreement with Model 1 from Dehnen & Binney (1998b), indicating a preference for a model where the disk, rather than the halo, dominates the circular speed at the solar neighborhood. The main uncertainty in this study is the underlying profile of the tracer populations, since this is something we cannot recover from our data due to the complicated selection function of the SDSS spectroscopic survey. Future unbiased spectroscopic surveys, such as those undertaken by the LAMOST telescope or the Gaia mission, will allow much more robust con-

straints to be made. A knowledge of the Galactic potential near the disk is clearly of great importance. In particular, it allows us to constrain the local dark matter density, which is a crucial piece of knowledge when evaluating the prospects for direct and indirect detection of dark matter (Bertone, Hooper & Silk 2005, and references therein). Despite slow progress since the classical work of Kuijken & Gilmore (1991), this field is set to have a great impact in the coming years.

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APPENDIX

THE TURN-OFF CORRECTION FOR THE PHOTOMETRIC PARALLAX RELATION

Our distances are estimated using the photometric parallax relation of Ivezić et al. (2008), with one minor modification regarding the treatment of the main-sequence turn-off. Ivezić et al. (2008) based their turn-off correction on the color-magnitude sequence for cluster M13, which has an age of around 10 Gyr and $[\text{Fe}/\text{H}] = -1.54$. Although this is a sound approach for stars belonging to the Galactic halo, disk stars will be (in general) both younger and more metal-rich than this cluster. Therefore we adopt an alternative approach to modelling the turn-off correction.

We follow the approach described in Smith et al. (2009a), where stellar models were used to construct a metallicity-dependent turn-off correction. Since we cannot discriminate between different populations on a stars-by-star basis, we must construct a global correction which reflects the relative numbers of thin-disk, thick-disk and halo stars at a given $[\text{Fe}/\text{H}]$. This is done by constructing a toy model to represent the properties of our sample, with parameters given in Table 4.

The final ingredient for this calculation are theoretical stellar models, which we take from Dotter et al. (2008). We downloaded three sets of models, corresponding to each of our three Galactic components. The prescription we adopt is the same as Smith et al. (2009a). In brief we shift each sequence so that it has $M_r = 0$ for $(g-i) = 0.6$ and then, for a given $[\text{Fe}/\text{H}]$, we calculate a weighted mean M_r as a function of $(g-i)$ in the range $0.3 < (g-i) < 0.6$, considering only model data up to the main-sequence turn-off. The weights are determined from the above toy model, so that we account for the contribution of each Galactic component at a given metallicity.

We then calculate the offset between the weighted mean model magnitude and the uncorrected relation of Ivezić et al. (2008), i.e. given in equations (A1–A5). This is shown in Fig. 10. Clearly, for metallicities below solar all of the models lie around $\Delta M_r^{\text{TO}} = 0$, i.e. they agree with the *uncorrected* parallax relation of Ivezić et al. (2008). Therefore we decide not to incorporate any turn-off correction when calculating our distances.

The only exception from this behavior is the solar-metallicity model, which is systematically offset by between 0.1 and 0.2 magnitudes. We believe this is due to problems with the normalization of the models (which we do by forcing all models to pass through the point $M_r = 0$ at $(g-i) = 0.6$). In any case, this should have little effect on our results as very few metal-rich stars are blue enough to lie in this turn-off region; only one per cent of our sample have $[\text{Fe}/\text{H}] > -0.25$ and $(g-i) < 0.6$.

There will be additional uncertainties on the parallax relation in this turn-off regime due to scatter in this correction. To account for this we calculate the standard deviation in M_r when we calculate the mean. The scatter varies as a function of color and metallicity, but if we take the relation with the largest scatter (corresponding to $[\text{Fe}/\text{H}] = -1$ dex) we find the following relation,

$$\delta(\Delta M_r^{\text{TO}}) = 0.272 - 0.454(g-i). \quad (\text{A1})$$

When estimating distances for our stars we add this uncertainty in quadrature to the other sources of error for stars in this color range.

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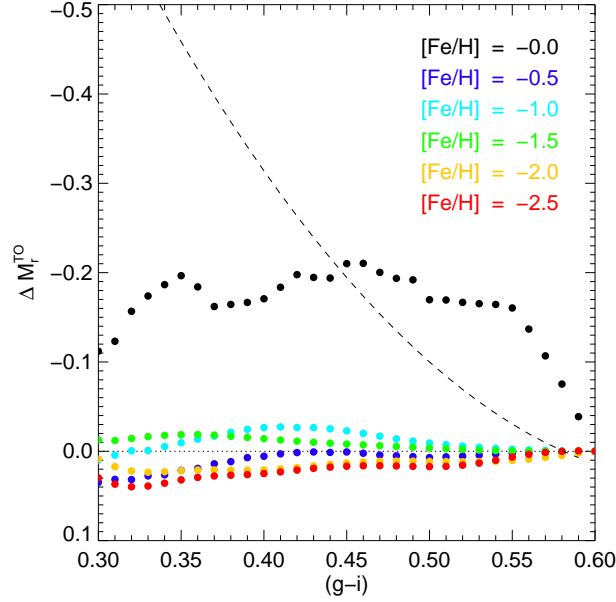


FIG. 10.— Our correction to the photometric parallax relation to account for main-sequence turn-off stars. The points correspond to the difference in magnitude between the stellar models and the parallax relation given in equations (A1–A5) of Ivezić et al. (2008). The dashed line denotes the turn-off correction given by Ivezić et al. (2008), which is based on the halo globular cluster M13.

TABLE 4

PARAMETERS OF THE TOY MODEL USED TO CONSTRUCT THE TURN-OFF CORRECTION TO THE PHOTOMETRIC PARALLAX RELATION. THE PARAMETERS ARE CHOSEN TO BE CONSISTENT WITH THE FOLLOWING REFERENCES: DENSITIES FROM JURIĆ ET AL (2008), WHICH WE EVALUATE AT $z = 1$ KPC, I.E. THE MEAN VALUE OF OUR SAMPLE; $[\text{Fe}/\text{H}]$ FROM SOUBIRAN ET AL. (2003) FOR THE DISK AND IVEZIĆ ET AL. (2008) FOR THE HALO; $[\alpha/\text{Fe}]$ FROM VENN ET AL. (2004); AGES FROM NORDSTRÖM ET AL. (2004) FOR THE DISK AND CHOSEN TO MATCH THE HALO AGE USED IN (SMITH ET AL. 2009A).

Component	Age (Gyr)		$[\text{Fe}/\text{H}]$ (dex)		$[\alpha/\text{Fe}]$ (dex)	ν ($z = -1$ kpc) ($\text{M}_\odot \cdot \text{pc}^{-3}$)
	Mean	Sigma	Mean	Sigma		
Thin Disk	2	1	-0.17	0.26	0.0	0.036
Thick Disk	8	4	-0.48	0.32	0.2	0.040
Halo	10	2	-1.46	0.3	0.3	0.005

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TABLE 1

KINEMATIC PROPERTIES OF THE DISC. THE VALUES OF σ_{Rz} AND α_{Rz} FOR STARS WITH $[\text{Fe}/\text{H}] < -0.8$ DEX ARE UNRELIABLE DUE TO THE DIFFICULTY IN MAKING THE HALO CORRECTION AND HENCE HAVE BEEN OMITTED FROM THIS TABLE. NOTE THAT σ_{Rz} AND α_{Rz} WERE CALCULATED USING ONLY STARS WITHIN $7.5 \text{ kpc} < R < 8.5 \text{ kpc}$ AND SO THE NUMBER USED IS LESS THAN THAT QUOTED IN THE SECOND COLUMN, ESPECIALLY AT LARGE z .

$[\text{Fe}/\text{H}]$ (dex)	$\langle z \rangle$ (kpc)	No. stars	Halo Frac.	$\langle v_R \rangle$ (km s^{-1})	$\langle v_\phi \rangle$ (km s^{-1})	$\langle v_z \rangle$ (km s^{-1})	σ_R (km s^{-1})	σ_ϕ (km s^{-1})	σ_z (km s^{-1})	σ_{Rz} ($\text{km}^2 \text{s}^{-2}$)	α_{Rz} ($^\circ$)	$\sigma_{R\phi}$ ($\text{km}^2 \text{s}^{-2}$)	$\alpha_{R\phi}$ ($^\circ$)
$(-0.5, +0.2)$	-0.59	736	0.00	$-0.0^{+1.6}_{-1.6}$	$-196.9^{+1.1}_{-1.1}$	$-2.7^{+0.9}_{-1.0}$	$38.8^{+1.2}_{-1.3}$	$27.7^{+0.4}_{-0.4}$	$22.4^{+0.7}_{-0.7}$	$-99.5^{+36.8}_{-36.8}$	$-5.6^{+2.1}_{-2.1}$	$110.4^{+45.6}_{-44.7}$	$8.3^{+3.4}_{-3.4}$
$(-0.5, +0.2)$	-0.86	736	0.00	$3.9^{+1.9}_{-1.8}$	$-188.4^{+1.2}_{-1.2}$	$-1.9^{+1.1}_{-1.1}$	$42.6^{+1.5}_{-1.4}$	$29.8^{+0.1}_{-0.1}$	$25.7^{+0.9}_{-0.9}$	$-322.5^{+66.6}_{-66.7}$	$-14.6^{+2.9}_{-2.8}$	$206.9^{+60.9}_{-61.5}$	$12.0^{+3.5}_{-3.4}$
$(-0.5, +0.2)$	-1.11	736	0.00	$5.6^{+1.9}_{-1.9}$	$-183.9^{+1.4}_{-1.4}$	$-6.0^{+1.2}_{-1.3}$	$43.5^{+1.6}_{-1.6}$	$33.6^{+1.1}_{-1.1}$	$28.5^{+1.0}_{-1.0}$	$-242.3^{+85.6}_{-86.5}$	$-12.0^{+4.1}_{-4.1}$	$100.3^{+76.8}_{-76.5}$	$7.4^{+5.5}_{-5.6}$
$(-0.5, +0.2)$	-1.53	736	0.02	$0.8^{+2.4}_{-2.4}$	$-179.3^{+1.6}_{-1.6}$	$-5.8^{+1.5}_{-1.4}$	$50.2^{+2.1}_{-2.2}$	$35.6^{+1.0}_{-0.9}$	$30.7^{+1.2}_{-1.2}$	$-362.9^{+104.2}_{-106.0}$	$-12.4^{+3.5}_{-3.5}$	$420.1^{+126.5}_{-127.1}$	$16.8^{+4.6}_{-4.6}$
$(-0.8, -0.5)$	-0.59	543	0.00	$0.5^{+2.0}_{-2.0}$	$-190.8^{+1.4}_{-1.5}$	$0.0^{+1.5}_{-1.5}$	$42.7^{+1.5}_{-1.6}$	$32.8^{+1.0}_{-1.0}$	$31.8^{+1.1}_{-1.1}$	$-193.6^{+71.9}_{-70.9}$	$-12.7^{+4.6}_{-4.6}$	$137.1^{+76.4}_{-75.3}$	$10.0^{+5.6}_{-5.5}$
$(-0.8, -0.5)$	-0.96	543	0.00	$6.1^{+2.4}_{-2.3}$	$-174.2^{+1.8}_{-1.8}$	$-1.8^{+1.8}_{-1.8}$	$48.2^{+1.9}_{-1.8}$	$39.6^{+1.5}_{-1.5}$	$38.8^{+1.4}_{-1.4}$	$-154.0^{+127.6}_{-127.8}$	$-10.2^{+8.3}_{-8.4}$	$-89.0^{+110.7}_{-110.3}$	$-6.6^{+8.3}_{-8.3}$
$(-0.8, -0.5)$	-1.26	543	0.01	$4.3^{+2.6}_{-2.5}$	$-168.6^{+2.1}_{-2.1}$	$-3.1^{+1.9}_{-1.9}$	$50.3^{+2.1}_{-2.1}$	$44.6^{+1.9}_{-1.9}$	$37.2^{+1.5}_{-1.5}$	$-181.0^{+133.7}_{-135.0}$	$-8.7^{+6.3}_{-6.4}$	$143.9^{+144.6}_{-144.0}$	$13.5^{+13.5}_{-13.5}$
$(-0.8, -0.5)$	-1.69	543	0.04	$4.0^{+3.2}_{-3.2}$	$-162.1^{+2.4}_{-2.4}$	$-5.0^{+2.2}_{-2.2}$	$57.6^{+2.7}_{-2.7}$	$44.2^{+2.4}_{-2.5}$	$39.6^{+1.8}_{-1.8}$	$-311.3^{+179.3}_{-179.8}$	$-9.7^{+5.6}_{-5.5}$	$519.6^{+227.6}_{-225.0}$	$18.3^{+7.3}_{-7.1}$
$(-1.5, -0.8)$	-0.60	541	0.11	$6.1^{+3.0}_{-2.9}$	$-177.5^{+2.1}_{-2.1}$	$0.6^{+2.2}_{-2.1}$	$52.9^{+2.7}_{-2.7}$	$40.8^{+2.0}_{-2.0}$	$40.2^{+1.8}_{-1.8}$	—	—	$133.6^{+418.9}_{-406.6}$	$5.2^{+17.7}_{-18.7}$
$(-1.5, -0.8)$	-0.99	541	0.14	$4.3^{+3.2}_{-3.3}$	$-160.6^{+2.6}_{-2.6}$	$0.1^{+2.6}_{-2.6}$	$53.0^{+3.2}_{-3.2}$	$42.6^{+2.6}_{-2.7}$	$46.1^{+2.1}_{-2.1}$	—	—	$-275.6^{+473.9}_{-471.1}$	$-10.7^{+21.9}_{-19.3}$
$(-1.5, -0.8)$	-1.32	541	0.17	$5.1^{+3.9}_{-4.0}$	$-150.4^{+3.3}_{-3.3}$	$1.0^{+2.8}_{-2.8}$	$61.3^{+4.1}_{-4.1}$	$50.7^{+2.7}_{-2.7}$	$45.7^{+2.4}_{-2.4}$	—	—	$359.8^{+555.2}_{-551.9}$	$11.8^{+18.9}_{-21.0}$
$(-1.5, -0.8)$	-1.73	541	0.27	$7.6^{+4.6}_{-4.7}$	$-134.5^{+4.0}_{-4.0}$	$1.2^{+3.4}_{-3.3}$	$60.2^{+4.6}_{-4.6}$	$55.3^{+3.0}_{-3.0}$	$45.4^{+3.0}_{-3.0}$	—	—	$-171.3^{+496.2}_{-492.8}$	$-8.7^{+36.8}_{-32.1}$